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# Power Transformer Digital Twin: Incorporating Thermodynamic and Water **Diffusion Discrete Elements Model for Enhanced Aging Calculation**

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### **SUMMARY**

The condition assessment of power transformers (i.e., active part) is mainly performed by testing the chemical and physical properties of the insulating liquid, and the dissolved gas content. Additional electrical tests are also performed using the accessible terminals, such as insulation resistance, power factor measurement, and dielectric frequency response (DFR). While a large amount of data can be extracted from these tests, they mostly represent average values of the complete set of windings/coils.

The digital twin concept presented in this paper includes chemistry, thermodynamic, and water diffusion equations, to model the most relevant processes happening within an energized transformer. It solves a system of equations for a series of discrete elements representing small portions of the windings, such a model allows the assessment of the local variation of properties and parameters over time, following the degradation processes endured by the materials.

The calculation starts from the known conditions of the insulating liquid and the coils after factory drying and tank filling with the dried insulating liquid. Utilizing the basic curves from the "Piper diagram", improved by the experiments from T. V. Oomen [1] [2], coupled with a transient representation of the diffusion process from Zhan et al [3], a system of equations is then generated, resembling a "finite elements" approach. The resultant localized values of moisture and temperature are in turn used for the calculation of the degradation rate of each discrete region of the solid insulation in the windings, using the equations from the latest version of the IEC 60076-7 [4]. The chemical representation of the rupture of the glycosidic bonds of the cellulose chain allows for the calculation of water generated due to the paper degradation. The additional water molecules are redistributed in the system during the next time step of the simulation.

# **KEYWORDS**

Transformer aging, Life Assessment, Digital Twin, Numerical Simulation.

### **INTRODUCTION**

Power transformers' testing techniques have evolved significantly in recent years. However, several parameters remain challenging to be directly measured. The two most relevant examples are the degradation rate and the water content of the insulating paper. While intrinsically correlated, such parameters are extremely complex to measure, not only due to the lack of a sensor but also due to their dynamic behavior and the challenge of identifying the most critical point. These two parameters are of paramount importance for the life assessment of a transformer. A more detailed discussion of such challenges can be found in state-of-the-art publications [6,7].

The development of the model presented in this article started with an analogy, comparing the measurement of such parameters with the positioning system of a nuclear submarine. In addition to being developed before the creation of the global positioning system, the navigation system of such submarines requires high accuracy. Additionally, they may remain in deep waters for prolonged periods. Thus, the solution was to register the initial position and track accurately every movement thereafter. By integrating the pathway, it was possible to define its current location. The same concept can be applied to the aforementioned parameters. Having a well-defined "initial point" and tracking continuously the variation with time, it is possible to tell with good accuracy the value at any instant of time.

The initial values of water content and mechanical strength (i.e., degree of polymerization) of the insulating paper can be measured in a new transformer. The drying process is usually finetuned to lead to 0.5% water content, and the nominal lifespan of the solid insulation system is set at 180,000 hours [8]. The challenge addressed by our approach is to accurately estimate the dynamic distribution of water inside the transformer and the equivalent life consumption for each time interval.

The estimation of water content in the materials inside the transformer will also include the estimation of water content in the insulating liquid. Since this happens to be a measurable parameter, it is possible to continuously adjust the estimated values, using a machine learning algorithm to ensure the convergence between measured and calculated values.

### **DEVELOPED MODEL**

The starting point for developing a thermo-chemical digital twin is to mathematically represent the thermo-chemical processes endured by a transformer, which is already a huge challenge in itself. A full set of core and coils of a transformer encompasses a couple hundred thousand pieces, from very small spacers to massive blocks of compressed cellulose. Thus, the fluid dynamics associated with heat transfer and electromagnetic phenomena cannot be represented to the last detail without requiring a computational effort to the level of the top-notch High-Performance Computational system (i.e., supercomputers). This is far from being the available hardware for an online monitoring system. Thus, several simplifications are required.

To handle the latter issue, the transformer is represented as a set of discrete elements. Such elements can be each spacer of a disc winding or a section of the same winding. A possible approach is to represent each coil as a sequence of vertical sections, assuming each one to have a homogeneous temperature and water content. A simplified version of the referred discrete elements is presented in Figure 1. While it can be far more refined, the simplified model used for this paper has two coils, which were represented as 5 sections, each corresponding to 20% of the discs. The thick insulation elements are relevant, so each of them was represented as one element (lower and upper stacks). The internal leads and the insulating fluid were the additional elements in this model. Further details and a graphical representation of the model can be found in [9]

The starting point to develop our model was the so-called "Piper Charts" named after the original work by John Piper in the 40's [10]. The chart currently used in some monitoring equipment for predicting the water content in the windings based on the water content in the mineral oil is a re-edition of the original, presenting isothermal equilibrium conditions for the water balance between the insulating paper and the mineral oil. The fundamental issues with using such equilibrium curves are the lack of equilibrium conditions and the large variation amidst the different elements made of cellulose inside a transformer.

The lack of equilibrium requires a two-fold approach, including both the dynamic behavior of the temperature distribution and the diffusion process of the water within the solid insulation. Using models from internal reports merged with the modeling proposed by Zahn [3], the diffusion process could be represented by a simplified geometry, the initial and ultimate water contents, and a time constant for the transient condition. Such an approach is very similar to the one used for the dynamic temperature distribution. The time constants allow the calculation of the exponential variation after a fraction of the time required for reaching equilibrium. When the time step is small enough, such a model's results are very close to those from the model described in the IEC 60076-7 [4] and others, such as Susa [11].

The model was developed considering the possibility of transformers filled with mineral oil, natural ester, and synthetic ester. For each case, it is necessary to have the design data corresponding to the fluid in use. While the thermal model will adjust the gradients according to loading and ambient temperature, it will not change the original design data.

# WATER BALANCE ISOTHERM CHARTS

While several papers and studies present the water balance isothermals for cellulose-based paper and mineral oil, converting the curves into a single equation is not trivial. Instead of fitting curves for the isothermals, as in Figure 1, our approach was to adjust curves for different relative water contents.



Figure 1 – Isothermal curves for the balance of water between paper and mineral oil.

A relevant challenge for the development of the model was the lack of information on the equilibrium curves for the water balance between paper and the fluid for ester liquids. The Cigre TB741 [12] was a relevant source of information, as well as the work from Zhang et al [13], [14], [15] and [16].



Figure 2 – Isothermal curves for the balance of water between paper and the natural ester liquid.

For natural ester liquids (Figure 2), the basic set of charts used came from the technical brochure of a brand of natural ester liquids which includes charts showing data on low water content very accurately as well as data on higher water contents [17]. A mathematical procedure was developed to allow the calculation of the water content on paper based on any given water content in the fluid and temperature.

In the case of synthetic esters, the most comprehensive piece of information was taken from [18] with the isothermal for temperatures between 0 and 80°C. Additional information for the higher temperatures was extracted from previously mentioned papers, leading to the curves presented in Figure 3.



Figure 3 – Isothermal curves for the balance of water between paper and the synthetic ester liquid.

#### WATER DIFFUSION MODEL

The water diffusion model was built based on a simplified geometry model allowing the calculation of a time constant for the variation of the water content in paper. The base equation was taken from [3].

$$D = D_0 * e^{\frac{C}{2} + E_a * \left(\frac{1}{T_0} - \frac{1}{T}\right)}$$
(1)

$$\tau = \frac{K.d^2}{\pi^2 D} \tag{2}$$

$$W_P = W_{P,i} + \left(W_{P,u} - W_{P,i}\right) * \left(1 - e^{\frac{\Delta t}{\tau}}\right)$$
(3)

Where:

*T*<sub>0</sub>, *E*<sub>a</sub>, *D*<sub>0</sub> = Constants *K* = geometry factor  $W_P, W_{P,i}, W_{P,u}$  = Water content in paper at a given time, *i* = initial, *u* = ultimate.

The inclusion of the transient diffusion model represented a huge step in the model. Without such curves, the estimated values of water content in the insulating liquid were unrealistic.

### PAPER AGING MODEL

Paper aging is calculated using three different approaches: the traditional curves based only on the temperature, the new equations from Annex A of [4] at a water content in paper fixed at 0.5%, and the same new equations using the value of water content calculated by the system.

Although the initial expectation was to find a good agreement level between the traditional Arrhenius curves (traditional model) and the new model at a constant 0.5% water content, the results were far from that. During the intervals where the temperature of the hottest spot sits in the lower range, the estimated aging factor using the new model gets to one order of magnitude higher than the traditional. Note that the chart in Figure 4 uses a log scale for the aging acceleration factor. Thus, even if keeping moisture content at 0.5% when using the new curve, there will be a significant reduction in the life expectancy for partially loaded transformers. Yet, the estimated paper life for partially loaded transformers exceeds one hundred years, what means that this failure mode will not be the most critical for transformer life. Thus, there is a potential for increasing the loading capacity of such units, as long as other potential risks associated with this are properly addressed.

While the table presented in Annex A of [4] compares the new curve with the traditional aging curve for non-upgraded paper insulation, the comparison with the most commonly used curve, that of Thermally Upgraded Paper (TUP) is included in Table 1 of [4] and shows the difference between them.



Figure 4 – Aging acceleration factor calculated per the traditional curve for TUP and the new model from [4] at a water content of 0.5% and free from air.

#### WATER GENERATED BY PAPER DEGRADATION

The scission of the oxygen bridge between two glucose rings generates water as a byproduct. Each scission of a cellulose molecule affects two glucose monomers (as represented in Fig. 5). Per the studies from [19], two of the hydroxyls are consumed in further steps of the reaction, leading to a "net" release of four molecules of water per cellulose chain scission. Each scission would lead to an average halving of the degree of polymerization (DP). As the number of molecules also doubles, it is unlikely that all the molecules will be affected simultaneously, resulting in the traditional exponential decay of the degree of polymerization.



Figure 5 – Chemical representation of two monomers of glucose (C6H10O5)n. The circled hydroxyls lead to water generation.



Figure 6 – Reproduction from [19], figure 9b. Total water production from the aging of Kraft at 130°C.

The complete degradation of a mass of cellulose requires approximately 8-10 scissions of the chain, also per the discussions in [19], which will be unevenly distributed among the cellulose chains and results in an average DP lower than 200 in contrast with the "theoretical value" of 4.68 that would be obtained when an initial value of 1200 is halved 8 times. The number of water molecules generated due to the degradation of the cellulose can be estimated considering the successive scissions using an exponential curve for the rate of generation. As per the literature (Fig. 9b from [19], reproduced in Fig 6 here), the measured effect is rather a linear curve. The adopted method in this model estimated the total water generation using the geometric progression and converted that into a rate of generation per equivalent aging hour. The exponential decay of the DP brings back the expected behavior.

The developed model resulted in a total water production of 68.4 mg/g when a paper having an initial DP of 1200 is subjected to 8 scissions. This value matches very closely the experimental results presented in [19], where the total water generation after 8 scissions is approximately 70mg/g.

### DISCRETE ELEMENTS INTEGRATED MODEL

As previously mentioned, presented in further detail in [10], the entire transformer is represented as a set of discrete elements, having all properties and variables calculated for each element. The coils are represented by a sequence of vertical sections. The lower and upper pressing rings (thick pressboard elements) are separate elements, which are added to the internal leads and the volume of insulating liquid completes the main discrete elements. A graphical representation is shown in Figure 7.

A key element is the integration time step. Starting from the "end of manufacturing" condition, where the water content in the paper and the insulating liquid is known, the first step represents the "settling time". Load is set to zero and temperature is set as constant during this period, neglecting small variations of ambient. In all cases, this period is characterized by a small increase in the water content in the paper and a relevant drop in the water content in the insulating liquid.



Figure 7 – Discrete elements representing a transformer.

The next steps start with the estimation of the new local temperatures based on the loading curve and ambient temperature. Then, a system of non-linear equations is solved to find a value of water content for each element that satisfies all the transient moisture balance conditions and the mass conservation for the system. We have implemented two mechanisms to solve the system of non-linear equations. Initially, we created a solver based on genetic algorithms (i.e., GA) where we created an initial population of possible solutions and defined the fitness function as the water content that most closely satisfies the moisture balance conditions. We then iterate through the selection, cross-over, and mutation phases until we find a water content value that is optimized based on a provided threshold. This implementation allows our solution to be portable and platform-independent. Alternatively, off-the-shelf we use an Python library scipy.optimize.fsolve which calculates the roots of the non-linear equations based on a modification of the Powell hybrid method [20]. The condition on the previous step is the initial guess for the solver, and a small enough integration step results in the system converging to a new slightly changed value.

Using the new model from [4] and the local values of water content and temperature, the paper degradation rate is calculated at the end of each time step. The water generated by the degradation is in turn added to the total water content in the transformer and redistributed amidst all discrete elements in the next iteration. The system of equations is repeated iteratively until the accumulated aging of the paper in any region reaches the limit value of 180,000 hours or when the simulation time exceeds a defined value. Such consideration takes into account that the end of life of the paper may not be the most critical failure mode in transformers that are lightly loaded. Since the most diligent maintenance procedure may not lead to a lifespan of other components of a transformer beyond a timeframe in the range of 60 years. Thus, even if the paper aging does not reach the accumulated hours for the end of life during a simulation time representing 60 years of continuous loading, the system will end the simulation.

#### COMPARISON OF THE SIMULATION FOR DIFFERENT INSULATING LIQUIDS

The developed model was applied to a "typical" duck curve, freely adjusted to reach a daily peak loading matching the transformer nameplate rating (1 pu – per unit), a minimum of 0.2 pu, and a daily average of 0.56 pu. The very same loading cycle is repeated every day, with no changes. The ambient temperature was based on the weather conditions in Raleigh, NC in the USA, adjusted for reaching a maximum of 40°C during the summer days and a yearly average of 20°C.

These conditions were applied to a theoretical 100MVA 34.5kV-230kV. This could represent either a step-up unit of a renewable generation park or a step-down substation transformer. The temperature gradients were adjusted to reach a hotspot temperature rise of 80K at rated conditions and the same gradients were used for HV and LV. While unrealistic for real-world conditions, the temperatures remained unchanged for the different insulating liquids, so the daily temperature variations were the

same throughout the year. The main objective of this paper is to evaluate the impacts of dynamic loading and ambient on moisture distribution and paper aging. Thus, the reached temperatures were the same for the three cases, presented in Figure 10.



The nominal life temperature for each insulation system was adjusted based on the current informative values from the Annex C of IEC 60076-14 and suggested by the producers of insulating materials. The insulating paper is thermally upgraded in all cases. When immersed in mineral oil the nominal life temperature was set as 110°C. For natural esters it was set at 130°C and for synthetic esters at 120°C. While the initial content of water in the paper was set to 0.5% in all cases, the content in the insulating liquid was set to 15 ppm in mineral oil, 60 ppm in the natural ester, and 100 ppm in the synthetic ester.



Figure 10 – Full-year distribution of temperatures for the insulating liquid and for the windings.

Based on the Piper charts for each fluid, the expected results were to have a drier paper in the case of the ester liquids. For the same water content in the liquid and the same temperature, the water content in the paper reduces following the sequence: Mineral oil  $\rightarrow$  Natural Ester  $\rightarrow$  Synthetic Ester. This would be true considering only insulating paper and the fluid under equilibrium conditions. The results of the simulation, however, lead to different results. Especially focusing on the water content in the hotspot, the highest water content during the first year of loading, when the generated water due to paper degradation was minimal, the windings in mineral oil resulted in values lower than those of ester liquids.

The highest resultant water content in the hotspot region was achieved by the natural ester-immersed unit. The differences were in the second decimal place of percentual units, almost negligible.

While initially counterintuitive, the higher saturation point of ester liquids seems to help transport the water from one component to another. The charts in Figure 11 show this behavior, as the range of variation of the water content in paper increased for the natural and synthetic esters in comparison to the mineral oil case. This is also true for the thick insulation components, which behave as "sponges" in all cases, retaining a lot of the water, whose average content does not oscillate as much as it does on paper.

Similar charts were generated for the full 60 years of simulation time, presented in Figure 12. As the transformer is considered hermetically sealed, the water generated by paper degradation is the only factor leading to an increase in the total mass of water inside the transformer over the years. In a real-world situation, it is possible that the transformers would be subjected to maintenance interventions for removing water, not included in the simulation model. The "increased mobility" of the water enabled by the ester liquids becomes even more clear in these charts.

The initial water content in the insulating liquid has a minor impact on the water distribution since more than 95% of the initial water content is in the solid insulation. It is extremely interesting to visualize how the numerical equations lead to trends in water content matching very well with the practical experience. The most important factor for the overall trend is, undoubtedly, the water balance curves.



a)

b)



c) Figure 11 – Variation of the water content in the insulating liquid and windings during the first year of loading. The curves show results for mineral oil (a), natural ester (b) and synthetic ester (c).

The water content in the mineral oil unit starts at 15ppm, drops to less than 2ppm after 24 hours of settling time at approximately 10°C homogeneous temperature (transformer energized with no load), and more than doubles after 60 years of uninterrupted operation. In the natural ester, the water content dropped from 60 ppm to approximately 45 ppm and from 100 ppm to 90 ppm in the case of synthetic ester. In all cases the water content in the solid insulation increased from 0.5% to 0.52%, confirming that the solid insulation "rules" the water content in such relatively short time variations.

When taking into perspective the full 60 years of simulation, the capacity of the liquids to hold more water starts playing an important role. The fluctuation of water content in the paper increases with the water saturation of each liquid. Each "cycle" of water content represents a one-year behavior, driven mostly by the temperature difference between winter and summer (average of 10°C during winter days and 35°C in summer), as the loading cycle is the same every day during the 60 years of simulation.



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Figure 12 – Variation of the water content in the insulating liquid and windings during the 60 years of loading for: (a) mineral oil, (b) natural ester and (c) synthetic ester immersed units.

The water content in the paper increases much more in the case of the mineral oil unit, where the liquid can't absorb much water. At the end of the simulation, the water content in the lower discs reaches 1.35% in the mineral oil unit, 0.88% in the natural ester, and 0.92% in the synthetic ester. In the hottest region of the coil, the values are respectively 0.7%, 0.51% and 0.54%. Again, the result is a consequence of the balance of water amidst the materials at different temperatures.

As the water content is taken into consideration for the Arrhenius curve, on top of the different values of nominal life temperature, the total degradation of the paper during the simulation time presented relevant variations. The charts in Figure 13 present the accumulated aging after 60 years of loading calculated per the new Annex A from [4] and per the traditional exponential curve. In all cases, the water generation was triggered by the degradation calculated per the new Annex A equation. The vertical axis do not use the same scale, as it would result in difficulties in visualizing the curves for natural ester.

As the loading fluctuates during the day between 0.2 and 1pu (average 0.56pu) and the ambient temperature is not constantly 30°C, the aging factor rarely sits at the unit. Thus, after 60 years of

simulation, the highest achieved value of paper aging was obtained by using the "traditional curve" for mineral oil, reaching about 20% of the total life. For the natural ester and synthetic ester cases, the accumulated life consumption reached 5.3% and 9.9% of the total life, respectively.





Conversely, the values per the new Annex A equation reached 12.4%, 1.8%, and 3.13% respectively. In all cases, the degradation of the paper will not be the determinant effect for the transformer's end of life. There is plenty of space for increasing the loading capacity of the transformer, given that the other collateral effects/risks are considered and mitigated.

#### Conclusion

While paper degradation is still considered the most relevant aspect of transformer life, there are additional failure modes to be considered. A transformer designed to reach 1pu max on the daily cycle and a yearly average loading of 0.56pu would be considered a medium to highly loaded unit by most experts. However, the effectively consumed life in the case of mineral oil was lower than 20% per the traditional model. The paper lifespan per the complete digital-twin model would exceed 160 years, meaning it is not the most probable failure mode. In the case of natural and synthetic ester units, the

resultant lifespan for the paper would exceed 1000 years and 650 years, respectively. This represents far too much margin on paper degradation.

In such cases, increasing the transformer *load-ability* brings relevant benefits and additional risks. It requires a proper design and selection of all components, taking in consideration the different stray fluxes, thermal excursions, etc. Moreover, the transformer assessment must watch very closely the potential risks of excessive water content in the insulation system. Water is the main enemy of the insulation system, and the risks are amplified when exploring the loading capacity beyond the nameplate rating. The use of online monitoring is essential, and the developed physical-chemical digital twin offers an additional level of accuracy to the model, of paramount importance for system reliability.

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